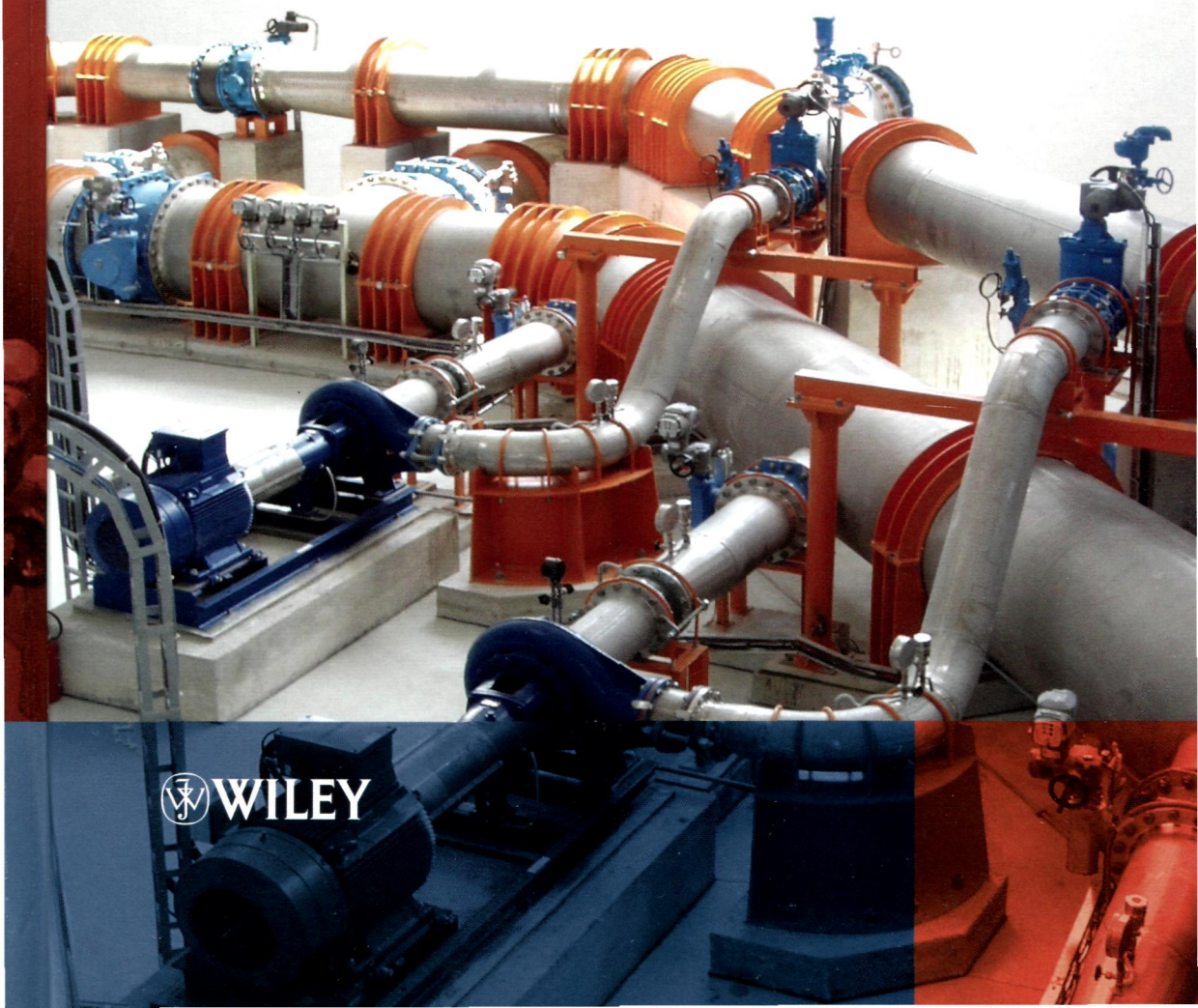
ANALYSIS AND MODELLING OF NON-STEADY FLOW IN PIPE AND CHANNEL NETWORKS

Vinko Jović



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