

Franz Durst

Fluid Mechanics

An Introduction to the
Theory of Fluid Flows

 Springer

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of Fluid Flows

With 347 Figures and 13 Tables



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